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A risk analysis model for radioactive wastes

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ABSTRACT

Hazardous wastes affect natural environmental systems to a significant extend, and therefore, it is necessary to control their harm through risk analysis. Herein, an effective risk methodology is proposed by considering their uncertain behaviors on stochastic, statistical and probabilistic bases. The basic element is attachment of a convenient probability distribution function (pdf) to a given waste quality measurement sequence. In this paper, ^{40}K contaminant measurements are adapted for risk assessment application after derivation of necessary fundamental formulations. The spatial contaminant distribution of ^{40}K is presented in the forms of maps and three-dimensional surfaces.

1. Introduction

In any modeling study, the primary source of information is the measurements. The risk analysis provides a common basis through measurements for improving the risk management in any society leading to objective decision-making [1]. The European Community (EC) prepares a report on this issue almost every year. Waste material risk analysis is important for environmental pollution control, and survival of living creatures. Although the destructive potential of risky accidental scenarios is, widely recognized, scarce attention is paid to this subject in the scientific and technical literature [2]. Quantitative Risk Assessment (QRA) is another method that is used in the chemical industry in land-use planning. There is also the risk analyses techniques that are use to prevent major hazard scenarios. Guldenmund et al. [3] proposed a management model to control barriers to prevent major hazard scenarios. An audit technique helps to assess the quality of such a management system.

Radioactivity has probabilistic and statistical nature. Diffusion of radioactive wastes, radiation and radionuclide has also a statistical nature. Among the potential radiological risks of wastes to human populations are the uses of gas concrete materials in buildings, industrial landfills of disposal solid wastes containing natural radionuclide. Among other risk assessment problems are decisions in nuclear emergencies, risk maps of radon-prone areas, modeling and improvement of a support system for protection after a nuclear accident [4–9].

In this study, a new risk analysis methodology is proposed, and its application is presented for ^{40}K wastes, which can be used for all the waste materials. Furthermore, the present study is an attempt to analyze risks and uncertainties of a radioactive waste in water reservoirs, especially when the underlying generating mechanism of the radionuclide concentrations is due to ^{40}K waste. It presents the risk assessment of radionuclide concentrations through the spatial distribution of ^{40}K waste measurements in water. In order to implement the methodology the Guarani aquifer data are adopted by Bonotto and Bueno [10], and the relevant risk inferences are presented in detail.

2. Methods

2.1. Risk analysis methodology for wastes

The simple risk, R, can be defined as the probability of occurrence of the ^{40}K waste variable, X, to be greater than the desired ^{40}K waste threshold, Q, at least once during a certain time duration, T, or over an area, A. In this paper, the area is the number of sampling points, n, for ^{40}K waste. If the sequence of future likely occurrence of X is $X_1, X_2, \ldots, X_n$ then the joint probability of non-occurrence, N, is defined as [11].

$$N = P(X \le Q) = P(X_1 \le Q, X_2 \le Q, \ldots, X_n \le Q) \quad (1)$$

Hence, the simple risk, R, as a complementary event can be defined as,

$$R = 1 - P(X_1 \le Q, X_2 \le Q, \ldots, X_n \le Q) \quad (2)$$

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Nomenclature

R	risk
T	return period
T_r	time between any two successive exceedences
μ	population mean
j	discrete duration of non-exceedence
α,β	parameters for probability distribution functions
p	probability of exceedence
X	random variable
x	any a variable
$P(\cdot)$	probability of the argument
$P(\vert)$	conditional probability of the argument
Q	design magnitude
Bq	Becquerel, unit of radioactivity
mBq/l	milli-Becquerel/liter

The calculation of the multivariate probability term on the right hand side of Eq. (1) is dependent on the structure of the variate considered and, in general, can be calculated by multiple integration of the multivariate probability distribution function (pdf) through tetra choric series' expansion [12,13].

In the risk assessment of any design project, it is necessary to decide first on the frequency of design ^{40}K waste occurrence, i.e., the return period, T, after which it is then possible to determine the magnitude of the design ^{40}K waste based on the most suitable pdf. The return period is the average length of time over which Q will exceed once. Hydro system engineers use the concept of the "return period" (or sometimes frequency of the occurrence or recurrence interval) as a substitute for probability because it gives some physical interpretation to the probability. The return period for a given event is the period on the "long-term average" over which a given event is equaled or exceeded. Hence, "on average", an event with a 5-year return period will be equaled or exceeded once in five years. The relationship between the probability and return period is given by

$$T = \frac{1}{P(X \geq Q)} = \frac{1}{1 - P(X < Q)} \tag{3}$$

in which Q is the value of the variate corresponding to a T-year return period. For example, if the probability that occurrence of the ^{40}K waste variable as we stated above will be equaled or exceeded in a single year is 0.2, that is, $P(X \geq Q) = 0.2$, then the corresponding return period is $1/P(X \geq Q) = 1/0.2 = 5$ years. Note that $P(X \geq Q)$ indicates the probability that the event is equaled or exceeded once over the return period, and it is the same for each year regardless of the magnitudes [14].

The random variable T_r which specifies the time or distance between any two successive exceedences of the design ^{40}K waste critical (threshold) is referred to as the waiting time. According to Feller [15] a sequence $\{a_r\}$ is a function defined for all positive integers; the binomial coefficient $\binom{x}{k}$ is a function defined for pairs of numbers (x, k) of which the second is a non-negative integer. In the same sense, one can say that the number T_r, of successes in r Bernoulli trials are a function defined on the sample space; to each of the 2^r points in this space there corresponds with a number T_r. A function defined on a sample space is a random variable.

Its distribution in the case of independent discrete observations at intervals, $j\,\Delta t$, is given by [13]. If interest lies in finding the probability, $P(T_r \geq j)$, of a positive run length to be greater than or equal to a given period, j, then the following risk analysis expression for any threat such as waste distribution and hydrologic cycle (floods, droughts, etc.) is given by [16–19] as

$$P(T_r \geq j) = P(j^+) + \sum_{k=1}^{\infty} P(k^-, j^+) \tag{4}$$

in which $P(j^+)$ = the probability that all the successive j variables are simultaneous positive; and $P(k^-, j^+)$ = the joint probability of simultaneous occurrence of j positives to be fallowed by k negative values. In any time series, there are simultaneous positive and negative sub-sections of different lengths in addition to negative (positive) sub-sections preceded and succeeded by different lengths of positive (negative) sub-sections. Any time series is a random composition of such sub-sections. It is important to note at this stage that $P(j^+)$ is a special form of $P(k^-, j^+)$ in which $k = 0$, and again Feller [15] and Şen [11] have also stated that

$$P(T_r = j) = P(T_r \geq j^+) - P(T_r \geq j + 1) \tag{5}$$

Generally, the computation of $P(k^-, j^+)$ can be completed through the multiple integration of the joint *pdf* of variables, $x_1, x_2, x_3, \ldots, x_{k+j}$, which can be written as

$$P(k^-, j^+) = \underbrace{\int_{-\infty}^{x_0} \cdots \int_{-\infty}^{x_0}}_{k} \underbrace{\int_{x_0}^{+\infty} \cdots \int_{x_0}^{+\infty}}_{j} f(x_1, x_2, x_3, \ldots, x_{k+j}) dx_1 dx_2 \ldots dx_{k+j} \tag{6}$$

in which $f(x_1, x_2, x_3, \ldots, x_{k+j})$ = multiple pdf. This integration is solved numerically [12] and through an analytical procedure [16]. The attractiveness of these formulations emerges first by considering their application to the normal independent process. In many practical applications, normal pdf plays a basic role and if the underlying pdf is not normal then the time series can be transformed to normally distributed case prior to the application of these methodologies. According to this method, the multivariate pdf in Eq. (6) is converted to one-dimensional multivariate pdf as follows

$$P(k^-, j^+) = \prod_{i=1}^{k} \int_{-\infty}^{x_0} f(x_i) dx_i \prod_{i=k+1}^{j+k} \int_{x_0}^{+\infty} f(x_i) dx_i \tag{7}$$

or in terms of the probabilities as.

$$P(k^-, j^+) = \prod_{i=1}^{k} P(x_i \leq x_0) \prod_{i=k+1}^{j+k} P(x_i > x_0) \tag{8}$$

Here, x_0 is the threshold level of the waste variable given. At the same time, the exceedence and non-excedence probabilities, p and q are,

$$q = P(x_i \leq x_0) = \int_{-\infty}^{x_0} f(x_i) dx_i \text{ and } p = 1 - q = P(x_i > x_0) = \int_{x_0}^{+\infty} f(x_i) dx_i \tag{9}$$

Substitution of these expressions into Eq. (7) produces

$$P(k^-, j^+) = \prod_{i=1}^{k} q \prod_{i=k+1}^{k+j} p = q^k p^j \tag{10}$$

when

$$k = 0; P\left(j^+\right) = p^j \tag{11}$$

Furthermore, by substituting Eqs. (10) and (11) into Eq. (4) the following equation can be obtained

$$P(T_r \geq j) = p^{j-1} \tag{12}$$

or from Eq. (5)

$$P(T_r = j)_{positive} = qp^{j-1} \tag{13}$$

Table 1
Theoretical distribution of the return period of an independent process as a function of the average return period T.

Average return period	Actual return period T_r exceeded various percentages of time or distance: $P(T_r \geq j)$						
T	0.01	0.05	0.25	0.50	0.75	0.95	0.99
2	7.64	5.32	3.00	2.00	1.41	1.07	1.01
5	21.64	14.42	7.21	4.10	2.28	1.23	1.04
10	44.71	28.43	14.16	7.58	3.73	1.48	1.09
30	136.84	89.36	41.89	21.44	9.48	2.51	1.29
100	459.21	299.07	138.93	69.97	29.62	6.10	2.00
1000	4603.86	2995.23	1386.60	692.80	288.53	52.53	11.11

In the same way, the probability, $P(T_r \geq j)$, of a negative run length (rareness of waste concentration), being equal to j, becomes

$$P(T_r = j)_{negative} = pq^{j-1} \tag{14}$$

where j is the discrete duration of non-exceedence. Hence, the return period of the stochastic process is the expected value of waiting time as,

$$E(T_r) = T = \sum_{j=1}^{\infty} jP(T_r = j) = p\sum_{j=1}^{\infty} j(1-p)^{j-1}$$
$$= p\left[1 + 2(1-p) + 3(1-p)^2 + \ldots\right]$$
$$= \frac{p}{[1-(1-p)]^2} = \frac{1}{p} \tag{15}$$

where $p = P(x > Q)$, i.e., the probability of exceedence of ^{40}K waste threshold value. Eq. (15) is the expected value of the geometric distribution.

Linsley et al. [20] have illustrated the theoretical distribution of the return period without any analytical expression. An error has been detected in Linsley et al. [20] about the actual return period T_r which has been assigned the value of zero. However, Gumbel [21] stated that the return period could not be less than one. The theoretical distribution of the return period is given by $P(T_r \geq j) = p^{j-1}$, the solution of which is presented in Table 1, where T_r is a function of $1 + \ln[P(T_r \geq j)]/\ln(1-(1/T))$. This table is the corrected form in Linsley et al. [20].

It can be seen from this table that over a long period or distance 25% of the intervals between waste concentrations greater than the 100 time (distance) units is less than about 30 time (distance) units while an equal number will be in excess of about 139 time (distance) units. In other words, for 75% safety that the desired threshold level will not be exceeded by a radionuclide within the next 30 time (distance) units, it must be designed for the 100 time (distance) units.

The risk of overtopping a given ^{40}K or potassium waste concentration threshold can be obtained in terms of the return period from Eq. (16) as

$$R = 1 - \left(1 - \frac{1}{T}\right)^n \tag{16}$$

Gupta [22] has provided the necessary tables and graphs for the application of Eq. (16) to engineering structures.

3. Results and discussion

3.1. Application of methodology

Potassium makes up 2.6% by weight of the Earth's crust. The element is enriched in acid magmatic rocks such as granite, containing potassium mica and potassium feldspar [23]. Potassium is hazardous due to its rapid reaction with moisture in mucous membranes and the skin [24]. ^{40}K, as one of the three natural isotopes, is a radioactive element that accounts for 0.012% of the total potassium, and has a half-life time of 1.35 billion years [25]. Therefore, due to very long half-life the determination of the ^{40}K concentration in a region is very important.

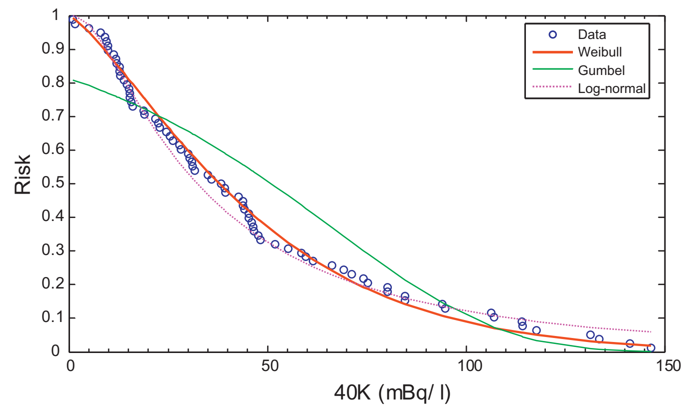

Fig. 1. Fitted probability distribution functions for ^{40}K waste measurements. It is seen that be the Weibull pdf of the most appropriate pdf to data in the three different pdf.

The fourth column in Table 2 indicates the natural ^{40}K radioactivity values from field measurements at 77 stations in the Guarani aquifer, Brazil [10]. These data are sorted from smallest to largest (the fifth column in Table 2) and are ranked (the sixth column in Table 2). Later, the risk (R) and return period (T) are computed. Exceedance probability in the ninth column is carried out with subtraction from "1" of the risk data.

A MATLAB® computer program is written to find the most appropriate three different pdf's for the data. These are Weibull, Gumbel and Lognormal pdf's, for which α and β parameters are obtained as shown in Table 3.

Fig. 1 indicates the best-fit pdf as Weibull for ^{40}K (mBq/l) measurements.

The theoretical Weibull pdf and the empirical frequency distribution from natural dissolved ^{40}K waste measurements in water (see Table 2) are presented in Fig. 2. It is obvious that a very good fit is valid between the histogram and the theoretical Weibull pdf.

The Weibull pdf is often used to model the time until an occurrence of an event where the probability of occurrence changes with time (the process has "memory"), in contrast to the exponential pdf where the probability of occurrence remains constant ("memoryless"). The theoretical Weibull pdf can be expressed in general as Eq. (17) and its cdf in Eq. (18) [26], Here x is the average value of ^{40}K (mBq/l) as given in Table 2.

$$f(x) = \alpha\beta^{-\alpha}x^{\alpha-1}\exp\left[-\left(\frac{x}{\beta}\right)^{\alpha}\right] \tag{17}$$

or succinctly as,

$$f(x) = 1 - \exp\left[-\left(\frac{x}{\beta}\right)^{\alpha}\right] \tag{18}$$

Table 2
Risk analysis and ^{40}K data (column 4 are adopted by Bonotto and Bueno [10]).

Stations (1)	Latitude (S) (2)	Longitude (W) (3)	^{40}K (mBq/l) (4)	Ordered ^{40}K (mBq/l) (5)	Rank (6)	[a]Risk (7)	Return period (1/R) (8)	[b]Exceedance of probability (9)	Occurence of Weibull probability (10)
1	23.1039	50.4022	15.4	1.100	1	1.2821	0.7800	0.9872	0.0413
2	23.2026	51.5519	43.9	1.600	2	2.5641	0.3900	0.9744	0.1178
3	23.1921	51.0949	11.9	5.000	3	3.8462	0.2600	0.9615	0.0319
4	22.2521	50.3338	15.2	8.000	4	5.1282	0.1950	0.9487	0.0408
5	21.4629	52.0527	38.4	9.100	5	6.4103	0.1560	0.9359	0.103
6	22.0645	51.2243	58.5	9.400	6	7.6923	0.1300	0.9231	0.157
7	21.1240	50.2621	12.7	9.900	7	8.9744	0.1114	0.9103	0.0341
8	21.1240	50.2621	13.0	9.900	8	10.2564	0.0975	0.8974	0.0349
9	22.0736	47.4100	22.6	11.300	9	11.5385	0.0867	0.8846	0.0606
10	22.1513	47.4905	39.2	11.900	10	12.8205	0.0780	0.8718	0.1052
11	22.1513	47.4905	5.00	12.100	11	14.1026	0.0709	0.8590	0.0134
12	20.1654	50.1432	18.8	12.700	12	15.3846	0.0650	0.8462	0.0505
13	20.1654	50.1432	21.8	12.700	13	16.6667	0.0600	0.8333	0.0585
14	22.0000	47.5338	51.9	13.000	14	17.9487	0.0557	0.8205	0.1393
15	22.0000	47.5338	46.1	13.800	15	19.2308	0.0520	0.8077	0.1237
16	20.4719	51.4149	1.60	14.600	16	20.5128	0.0488	0.7949	0.0043
17	20.1654	50.3338	22.9	15.200	17	21.7949	0.0459	0.7821	0.0615
18	20.4357	48.5433	9.10	15.400	18	23.0769	0.0433	0.7692	0.0244
19	20.4719	49.2244	12.1	15.400	19	24.3590	0.0411	0.7564	0.0325
20	20.4719	49.2244	11.3	15.700	20	25.6410	0.0390	0.7436	0.0303
21	21.0736	48.5905	8.0	16.000	21	26.9231	0.0371	0.7308	0.0215
22	21.2753	49.1433	15.7	18.800	22	28.2051	0.0355	0.7179	0.0421
23	21.3439	48.4905	80.3	19.000	23	29.4872	0.0339	0.7051	0.2155
24	21.4357	48.5000	16.0	21.800	24	30.7692	0.0325	0.6923	0.0429
25	21.3943	49.4433	1.10	22.600	25	32.0513	0.0312	0.6795	0.003
26	21.3943	49.4433	14.6	22.900	26	33.3333	0.0300	0.6667	0.0392
27	22.1835	49.0527	25.4	24.600	27	34.6154	0.0289	0.6538	0.0682
28	22.2430	49.0811	35.9	25.400	28	35.8974	0.0279	0.6410	0.0963
29	22.2108	48.4716	131.4	26.200	29	37.1795	0.0269	0.6282	0.3526
30	22.2521	48.4716	74.2	27.900	30	38.4615	0.0260	0.6154	0.0413
31	22.5224	48.2716	66.2	28.200	31	39.7436	0.0252	0.6026	0.1991
32	22.3207	47.5433	19.0	30.100	32	41.0256	0.0244	0.5897	0.1776
33	21.2844	47.3433	69.3	30.400	33	42.3077	0.0236	0.5769	0.051
34	21.1745	47.3338	30.9	30.900	34	43.5897	0.0229	0.5641	0.186
35	21.1241	47.3811	45.3	31.200	35	44.8718	0.0223	0.5513	0.0829
36	20.5224	47.3716	114.0	31.700	36	46.1538	0.0217	0.5385	0.1216
37	21.1008	47.4905	141.3	35.000	37	47.4359	0.0211	0.5256	0.3059
38	21.1008	47.4905	106.3	35.900	38	48.7179	0.0205	0.5128	0.3791
39	21.1008	47.4905	146.6	38.400	39	50.0000	0.0200	0.5000	0.2852
40	21.0736	47.5905	117.8	39.200	40	51.2821	0.0195	0.4872	0.3933
41	21.1422	48.1905	9.40	39.500	41	52.5641	0.0190	0.4744	0.3161
42	21.1513	48.3000	61.5	42.800	42	53.8462	0.0186	0.4615	0.0252
43	21.2108	48.1433	71.2	43.900	43	55.1282	0.0181	0.4487	0.165
44	21.3530	48.2244	133.6	43.900	44	56.4103	0.0177	0.4359	0.1911
45	21.2935	48.0244	94.10	44.200	45	57.6923	0.0173	0.4231	0.3585
46	21.4357	48.0622	94.7	45.300	46	58.9744	0.0170	0.4103	0.2525
47	20.4629	48.1000	80.3	45.300	47	60.2564	0.0166	0.3974	0.2541
48	21.5728	48.0149	114.3	46.100	48	61.5385	0.0162	0.3846	0.2155
49	21.5405	47.3811	27.9	46.400	49	62.8205	0.0159	0.3718	0.3067
50	21.5909	48.2622	13.8	46.600	50	64.1026	0.0156	0.3590	0.0749
51	22.0827	48.3054	59.6	47.700	51	65.3846	0.0153	0.3462	0.037
52	22.0323	48.4527	107.1	48.300	52	66.6667	0.0150	0.3333	0.1599
53	22.2844	48.3433	46.6	51.900	53	67.9487	0.0147	0.3205	0.2874
54	22.5456	49.3905	15.4	55.200	54	69.2308	0.0144	0.3077	0.125
55	23.0141	49.2905	30.1	58.500	55	70.5128	0.0142	0.2949	0.0413
56	23.1603	49.2905	75.3	59.600	56	71.7949	0.0139	0.2821	0.0808
57	22.5224	49.1338	84.7	61.500	57	73.0769	0.0137	0.2692	0.2021
58	23.0646	48.5433	35.0	66.200	58	74.3590	0.0134	0.2564	0.2273
59	23.0646	48.5433	42.8	69.300	59	75.6410	0.0132	0.2436	0.0939
60	22.2702	49.0000	48.3	71.200	60	76.9231	0.0130	0.2308	0.1149
61	22.1654	48.3338	39.5	74.200	61	78.2051	0.0128	0.2179	0.1296
62	23.1822	50.1917	47.7	75.300	62	79.4872	0.0126	0.2051	0.106
63	23.2050	50.2019	24.6	80.300	63	80.7692	0.0124	0.1923	0.128
64	23.0233	50.0410	26.2	80.300	64	82.0513	0.0122	0.1795	0.066
65	27.1430	52.0200	9.9	84.700	65	83.3333	0.0120	0.1667	0.0703
66	27.1800	50.2600	45.3	84.700	66	84.6154	0.0118	0.1538	0.0266
67	29.2637	51.1750	12.7	94.100	67	85.8974	0.0116	0.1410	0.1216
68	29.2333	51.5659	44.2	94.700	68	87.1795	0.0115	0.1282	0.0341
69	29.3244	55.0730	9.9	106.300	69	88.4615	0.0113	0.1154	0.1186
70	29.4716	55.4611	31.7	107.100	70	89.7436	0.0111	0.1026	0.0266
71	29.5649	56.3729	55.2	114.000	71	91.0256	0.0110	0.0897	0.0851
72	29.5649	56.3729	31.2	114.300	72	92.3077	0.0108	0.0769	0.1481
73	23.0350	55.1521	30.4	117.800	73	93.5897	0.0107	0.0641	0.0837

Table 2 (*Continued*)

Stations (1)	Latitude (S) (2)	Longitude (W) (3)	^{40}K (mBq/l) (4)	Ordered ^{40}K (mBq/l) (5)	Rank (6)	[a]Risk (7)	Return period (1/*R*) (8)	[b]Exceedance of probability (9)	Occurence of Weibull probability (10)
74	22.1300	54.5000	46.4	131.400	74	94.8718	0.0105	0.0513	0.0816
75	21.3934	55.0946	43.9	133.600	75	96.1538	0.0104	0.0385	0.1245
76	20.5609	54.5807	28.2	141.300	76	97.4359	0.0103	0.0256	0.1178
77	20.2613	54.3913	84.7	146.600	77	98.7179	0.0101	0.0128	0.0757

[a] Risk = $m/n+1$; m : rank, n : number of samples.
[b] 1 − Risk.

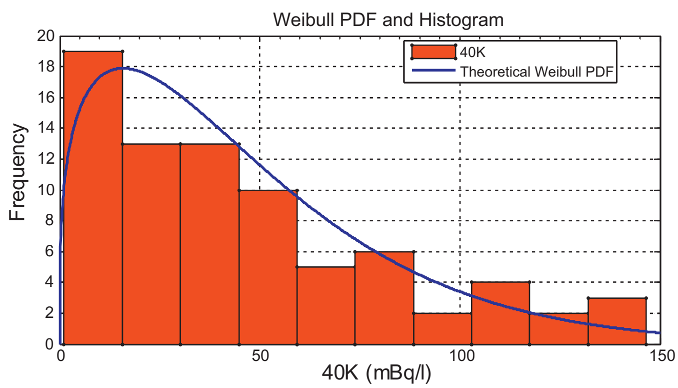

Fig. 2. Frequency distribution of ^{40}K and theoretical Weibull probability distribution function.

Table 3
α and β parameters for probability distribution functions.

Probability distribution functions	Parameter	
	α	β
Weibull	50.4394	1.28960
Gumbel	66.4421	42.2304
Log-normal	3.47790	0.965700

where β is the shape parameter, also known as the Weibull slope and α is the scale parameter. If α and β parameters from Table 3 are inserted into these equations then, the final Weibull pdf and cdf can be obtained as follows:

$$f(x) = 50.4394 \times 1.2896^{-50.4394} x^{50.4394-1} \exp\left[-\left(\frac{x}{1.2896}\right)^{50.4394}\right] \quad (19)$$

and

$$f(x) = 1 - \exp\left[-\left(\frac{x}{1.2896}\right)^{50.4394}\right] \quad (20)$$

Respectively, the graphical representations of these expressions are given in Fig. 3a and b.

Fig. 3a shows the change of probability of occurrence as less than any given threshold value on the horizontal axis. By definition, the probability of non-occurrence is a complementary value to the probability of occurrence (see Fig. 3b). The probability of each ^{40}K (mBq/l) value (the last column in Table 2) can be calculated from Eqs. (17) and (18).

The risk model presented by using the Weibull distribution in this research can be used in reliability and life data analysis due to its versatility. An important aspect of the Weibull distribution is how the values of the shape parameter, β (in Table 3), and the scale parameter, α (in Table 3), affect such distribution characteristics as the shape of the pdf curve, the reliability and the failure rate. The Weibull shape parameter, β, is also known as the Weibull slope. This is because the value of β is equal to the slope of the line on the probability plot. Different values of the shape parameter can mark effects on the behavior of the distribution. For example, when $\beta = 1$, the pdf of the three-parameter Weibull reduces to that of the two-parameter exponential distribution. The parameter β is a pure number, i.e. it is dimensionless [27].

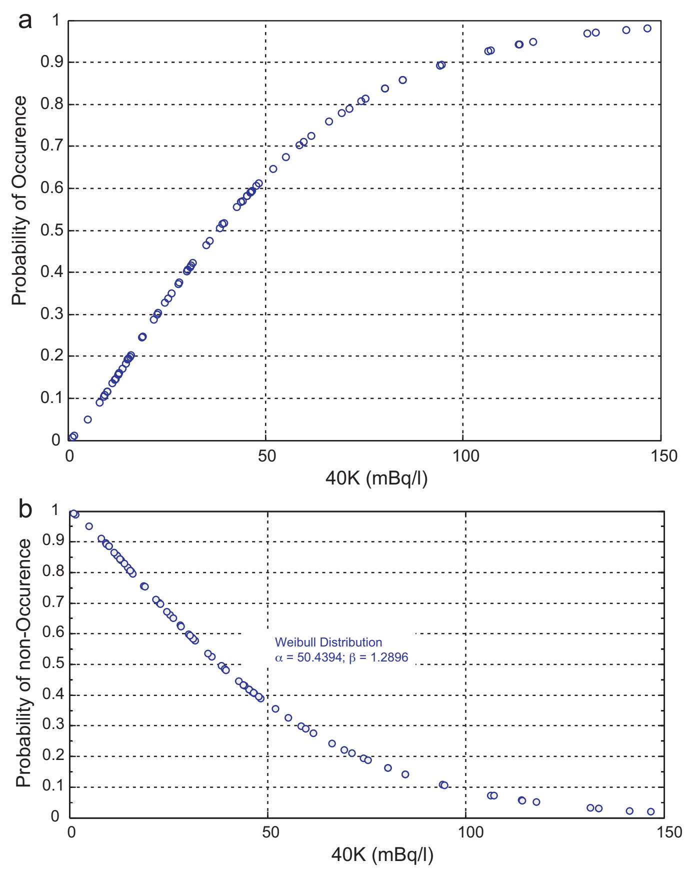

Fig. 3. (a) Weibull cumulative probability function for ^{40}K. (b) Probability of non-occurrence for ^{40}K.

The change of β's size also is one of the most important aspects of the effect of β on the Weibull distribution. Weibull distributions with $\beta < 1$ have an acceptable failure rate that decreases with time, also known as early-life failures. Weibull distributions with β close to or equal to 1 have a fairly constant failure rate, indicative of useful life or random failures. Weibull distributions with $\beta > 1$ (as in this research) have a failure rate that increases with time, also known as wear-out failures. These comprise the three sections of the classic "bathtub curve" [27]. A mixed Weibull distribution with sub-populations parameter values as $\beta < 1$, $\beta = 1$ and $\beta > 1$ would have a failure rate plot that is identical to the bathtub curve. An example of a bathtub curve is shown in Fig. 4. Fig. 3a and b completely comply with Fig. 4.

Observation of spatial distribution of ^{40}K in the research area is very important for controlling the potassium wastes. The waste contributions incoming from all the sampling stations must be taken into account for reliable results. Therefore, for the spatial distribution model of ^{40}K a waste distribution map is generated based

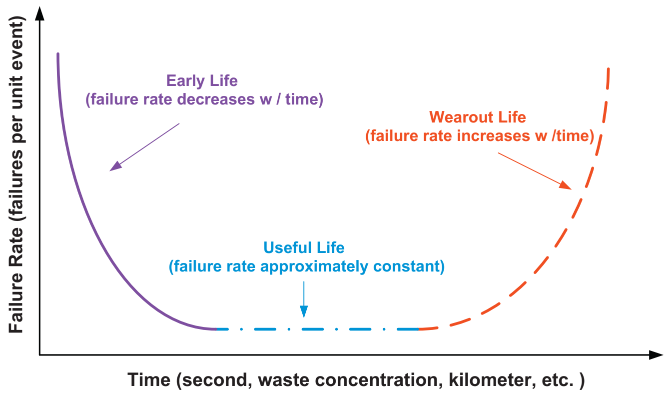

Fig. 4. An example the bathtub curve. The curve consists from three parts for $\beta < 1$, $\beta = 1$ and $\beta > 1$ situations.

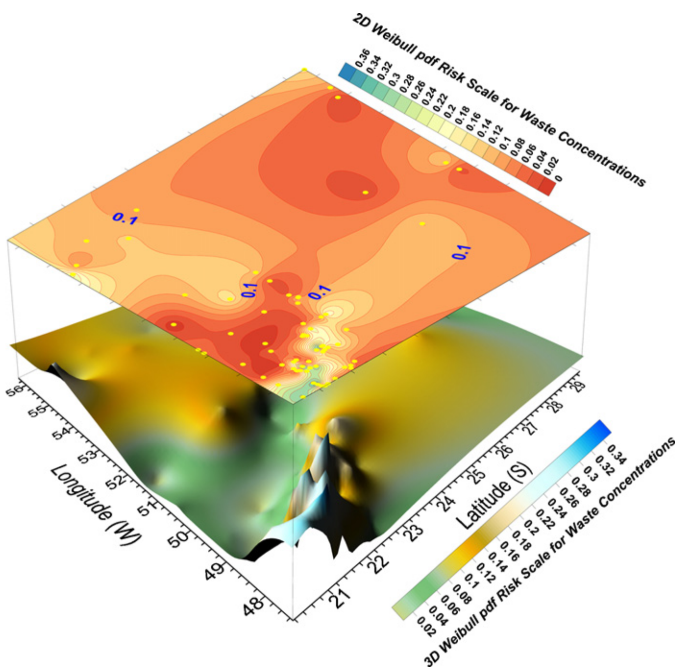

Fig. 5. Sampling points and ^{40}K risk distribution formed according to Weibull pdf. Sampling stations are the yellow points. The upper part is 2D and shows the iso-Weibull pdf risk distribution. The lower figure shows distribution of 3D Weibull pdf. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

on the Kriging methodology. The details about Kriging and variogram methodology are already widely given in literature [28,29]. The occurrence of Weibull probability values in the last column in Table 2 is used to draw Kriging map. The spatial risk distribution model of these probabilities is shown in Fig. 5 together with the sample locations (the yellow points). Fig. 5 consists of two main parts as the upper and the lower portions. The upper portion has 2-dimension (2D) and shows the iso-Weibull risk distribution lines. In the bottom left portion of the research area, the probabilities change rather randomly at long distances, whereas towards the upper side they become more regular. ^{40}K activity in these sections shows a greater the waste risk distribution than other parts.

The lower part in Fig. 5 is in 3-dimension (3D) and has a different point of view for distribution of the waste risk. The 3D part is also random, and it is clearly seen that high-risk regions in terms of radioactivity in the bottom left part of research area as in the 2D part. The high-risk regions in terms of radioactivity have the sedimentary basin. According to Bonotto and Bueno [10], the sedimentary sequence is almost undisturbed, with gentle dips towards the center of the basin. Separately, the region has the major stratigraphics basin units, where there are sandstones, conglomerates, diamictites, siltstones, shale mudstones, limestone, basalt and diabase. These forms based on sedimentary structures constitute clay minerals. Clay minerals are very common in fine-grained sedimentary rocks such as shale, mudstone and siltstone and in fine-grained metamorphic slate. Potassium is adsorbed rapidly in clay minerals [10]. Furthermore, the region has the basalt relicts. Basalt contains the high rate of ^{40}K and potassium. These structures create regions of high-risk distribution in Fig. 5.

4. Conclusions

Risk analysis methods and assessments developed in this research can be used for any waste materials. Herein, a risk analysis research is presented for natural dissolved ^{40}K waste distributions in the aquatic environment. According to this investigation, some methodologies that are widely used and even still in use in hydrology might be also used with success for risk analysis of any waste materials in the environment. To see the spatial variation of risk levels, different sampling locations are presented through 2D and 3D maps based on the theoretical Weibull probability distribution function (pdf) of each concerned hazardous material. Observation of the spatial variations has further strengthened the interpretation of the results. The Weibull model and the risk analysis in this research explain successfully spatial distribution of potassium in the environmental.Waste materials have been a significant concern for the habitat. Without sufficient control and mitigation of waste materials, reactive incidents have led to severe consequences, such as the release of radioactive waste and contaminated materials, radiation sickness, and threats to human lives, properties, and the environment. Consequences of waste materials can be well understood through risk analysis, risk assessment and computational techniques.

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